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ANALYTICAL INVESTIGATION OF THE SOLIDIFICATION  
PROCESS OF A LIQUID METAL IN CONTINUOUS CASTING  
UNITS

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We consider the fully established process of the solidification of a flat continuous ingot in a cooling system with a liquid-metal heat-transfer medium, filling the gap between the surface of the ingot and the water-cooled wall of the crystallizer (Fig. 1, where 1 is the ingot; 2 is the liquid-metal heat-transfer medium; 3 is the wall; 4 is the cooling medium; and 5 is a capillary packing). Here we shall assume that the external water cooling can be regulated along the ingot, for example, by sectional heat removal. The presence of a liquid-metal heat-transfer medium between the surface of the ingot and the water-cooled wall excludes the formation of a gas gap, which makes it possible to increase the rate of the cooling process, making it uniform around the perimeter of the ingot.

We assume that the transfer of heat along the Z axis due to thermal conductivity can be neglected in comparison with convective heat transfer [1] and that the temperature of the metal in the liquid phase is equal to the crystallization temperature. Under these conditions, we shall take account of the effect of heating of the melt by a corresponding increase in the latent heat of fusion in the approximation of the Stefan condition.

§1. If the width of the ingot is much greater than its thickness, then the solution of the problem posed will depend only on the two variables x and z. We select a Cartesian system of coordinates with the Z axis lying in the plane of symmetry of the ingot and as the origin of coordinates we take the point of intersection of the Z axis with a plane passing through the point of the start of crystallization. Taking account of the assumptions made above, the equation determining the temperature distribution in the solid phase has the form

$$\rho v C \frac{\partial T}{\partial z} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right), \quad (1.1)$$

where v is the velocity;  $\rho$  is the density; C is the heat capacity; and  $\lambda$  is the thermal conductivity of the ingot.

We write the boundary condition at the surface of the ingot in the form of the Newton-Richman law

$$\lambda \frac{\partial T}{\partial x} \Big|_{x=x_0} = -k(T|_{x=x_0} - T_M(z)), \quad (1.2)$$

where  $2x_0$  is the thickness of the ingot;  $T_M$  is the temperature of the cooling medium (water), which is assumed to be a given function of the coordinate z;  $k = (R_H + R_W + R_r)$  is the heat-transfer coefficient;  $R_H$  and  $R_W$  are the thermal resistances of the liquid-metal heat-transfer medium and the wall; and  $R_r$  is the external heat resistance.

At the crystallization surface, the following conditions must be observed:

$$\lambda \frac{\partial T}{\partial x} \Big|_{x=\xi(z)} = \kappa^* \rho_c v \xi'(z); \quad (1.3)$$

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$$T|_{x=\xi(z)} = T_c; \quad (1.4)$$

$$\xi(0) = x_0, \quad \xi(h) = 0, \quad (1.5)$$

where  $\kappa^* = \kappa + \frac{1}{2} \frac{(\rho_1 C_1)_m}{\rho_c} \Delta T_{\text{sup}}$ ,  $\kappa$  is the latent heat of fusion;  $\Delta T_{\text{sup}}$  is the superheat of the melt;  $(\rho_1 C_1)_m$

is the mean value of the product of the density by the heat capacity of the liquid phase;  $T_c$  is the crystallization temperature;  $\rho_c$  is the density of the solid phase with  $T = T_c$ ;  $\xi(z)$  is the crystallization surface;  $h$  is the depth of the liquid crater [the prime in (1.3) means differentiation with respect to  $z$ ].

Going over to the Kirchhoff variable

$$\Theta = \int_{T_c}^T \lambda(T) dT, \quad (1.6)$$

we write Eqs. (1.1)-(1.5) in the form

$$\frac{\nu \rho C}{\lambda} \frac{\partial \Theta}{\partial z} = \frac{\partial^2 \Theta}{\partial x^2}; \quad (1.7)$$

$$\frac{\partial \Theta}{\partial x} \Big|_{x=x_0} = -k \left( \frac{1}{\lambda_m} \Theta \Big|_{x=x_0} + T_c - T_M \right); \quad (1.8)$$

$$\frac{\partial \Theta}{\partial x} \Big|_{x=\xi(z)} = \kappa^* \rho_c \nu \xi'(z); \quad (1.9)$$

$$\Theta|_{x=\xi(z)} = 0; \quad (1.10)$$

$$\xi(0) = x_0, \quad \xi(h) = 0. \quad (1.11)$$

Condition (1.8) was obtained by linearization of relationship (1.2). For this purpose, the thermal conductivity  $\lambda(T)$ , standing under the integral sign in (1.6), was replaced by its mean value  $\lambda_m$  in the interval of temperature change under consideration. Integrating (1.6) for  $\lambda(T) = \lambda_m$ , we find the value of  $\Theta$  at the point  $x = x_0$ :

$$\Theta|_{x=x_0} = \lambda_m (T|_{x=x_0} - T_c).$$

Expressing the temperature  $T|_{x=x_0}$  from this, and substituting its value into (1.2), we obtain expression (1.8).

For some metals, the thermal diffusivity  $\lambda/\rho C = a$  is a weaker function of the temperature than that determining its thermophysical quantities  $\rho$ ,  $C$ , and  $\lambda$ ; therefore, with a sufficient degree of accuracy, it can be regarded as constant. In the general case we shall set

$$a_m = \lambda/\rho C,$$

where  $a_m$  is the mean value of the thermal diffusivity.

We bring Eqs. (1.6)-(1.11) into dimensionless form, taking as characteristic quantities  $x_0$ ,  $T_c$ ,  $\lambda_c$ ,  $C_c$ , where  $\lambda_c$  and  $C_c$  are the thermal conductivity and the heat capacity, respectively, for  $T = T_c$ . Denoting dimensionless quantities by a stroke above, we obtain

$$\bar{\Theta} = \int_1^{\bar{T}} \bar{\lambda}(\bar{T}) d\bar{T}; \quad (1.12)$$

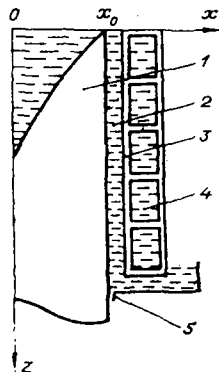


Fig. 1

$$\text{Pe} \frac{\partial \bar{\Theta}}{\partial z} = a_m \frac{\partial^2 \bar{\Theta}}{\partial x^2}; \quad (1.13)$$

$$\left. \frac{\partial \bar{\Theta}}{\partial x} \right|_{x=1} = -\text{Bi} \left( \frac{1}{\lambda_m} \bar{\Theta} \Big|_{x=1} + 1 - \bar{T}_M \right); \quad (1.14)$$

$$\left. \frac{\partial \bar{\Theta}}{\partial x} \right|_{x=\xi} = \bar{x}^* \text{Pe} \bar{\xi}'(\bar{z}); \quad (1.15)$$

$$\bar{\Theta} \Big|_{x=\xi} = 0; \quad (1.16)$$

$$\bar{\xi}(0) = 1, \quad \bar{\xi}(\bar{h}) = 0, \quad (1.17)$$

where  $\text{Pe} = \frac{\eta_c C_c z_0}{\lambda_c}$  is the Péclet number;  $\text{Bi} = \frac{k x_0}{\lambda_c}$  is the Biot number.

The solution of Eq. (1.13) under the conditions (1.15), (1.16) can be represented in the form of an expansion in series analogously to [2]:

$$\Theta(x, z) = -\kappa^* \text{Pe} \sum_{i=0}^{\infty} \left( \frac{\text{Pe}}{a_m} \right)^i \frac{1}{(2i+2)!} \frac{\partial^{i+1}}{\partial z^{i+1}} [x - \xi(z)]^{2i+2}. \quad (1.18)$$

Here and in what follows the stroke above dimensionless quantities is omitted.

From relationship (1.18), taking account of condition (1.14), for determining the function  $\xi(x)$  we obtain the equation

$$\sum_{i=1}^{\infty} \left( \frac{\text{Pe}}{a_m} \right)^i \left[ \frac{1}{(2i-1)!} \frac{d^i}{dz^i} (1-\xi)^{2i-1} + \frac{\text{Bi}}{\lambda_m} \frac{1}{(2i)!} \frac{d^i}{dz^i} (1-\xi)^{2i} \right] = \frac{\text{Bi}}{\kappa^* a_m} (1 - T_M). \quad (1.19)$$

For a given value of the function  $T_M$ , Eq. (1.19) must be integrated taking the conditions (1.17) into consideration.

The obtained equation (1.19) can be used conveniently for solution of the inverse problem (finding the law of the cooling of the ingot) for a given form of the crystallization surface  $\xi(z)$  and a given depth of the liquid crater  $h$ .

We shall carry out the investigation for  $\text{Bi} = \text{const}$  and  $T_M = \text{const}$ . In this case, limiting ourselves to the first term in (1.19), taking account of the first condition of (1.17), we find

$$\xi(z) = 1 + \frac{\lambda_m}{\text{Bi}} - \sqrt{\frac{2\lambda_m \Delta T}{\kappa^* \text{Pe}} z + \frac{\lambda_m^2}{\text{Bi}^2}}, \quad (1.20)$$

where  $\Delta T = 1 - T_M$ , or, returning to dimensionless quantities, we write (1.20) in the form

$$\xi(z) = x_0 - \frac{\lambda_m}{k} - \sqrt{\frac{2\lambda_m \Delta T}{\kappa^* \rho_c \nu} z + \frac{\lambda_m^2}{k^2}}. \quad (1.21)$$

For an infinitely high rate of heat removal ( $k \rightarrow \infty$ ), expression (1.21) coincides in form with the well-known "square-root" law for the Stefan problem [3]:

$$\xi_1(z) = x_0 - \xi(z) = \beta \sqrt{z},$$

where  $\xi_1(z)$  is the thickness of the solid phase;  $\beta = (2\lambda_m \Delta T / \kappa^* \rho_c \nu)^{1/2}$ . Satisfying (1.20) and the second condition of (1.17), we find the depth of the liquid crater

$$h = \frac{(1 + 2\lambda_m / \text{Bi}) \kappa^* \text{Pe}}{2\lambda_m \Delta T},$$

or, in dimensional quantities,

$$h = (1/2) A (1 + 2b/x_0) x_0^2 \nu, \quad (1.22)$$

$$A = \lambda_m \Delta T / \kappa^* \rho_c, \quad b = \lambda_m / k.$$

From this it is apparent that the value of  $h$  is directly proportional to the rate of pulling of the ingot and, for  $x_0/2b \gg 1$ , varies as the square of the thickness of the ingot. In the case  $x_0/2b \ll 1$  (a weak rate of heat removal), the depth of the liquid crater depends linearly on the thickness of the ingot.

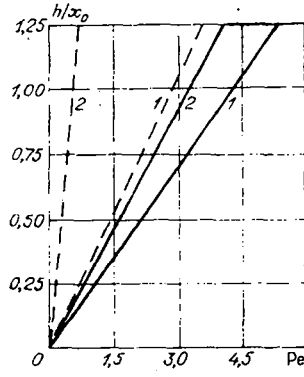


Fig. 2

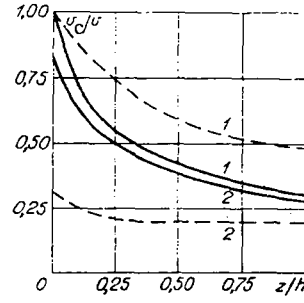


Fig. 3

The rate of crystallization  $v_c$  is connected with the rate of pulling  $v$  by the relationship [4]

$$v_c = v \sin \varphi,$$

where  $\varphi$  is the angle between the direction of the tangent at some point of the crystallization surface and the  $z$  axis. Since  $\sin \varphi = -\xi'(z)\{1 + [\xi'(z)]^2\}^{-1/2}$ , then

$$v_c = -v\xi'(z)\{1 + [\xi'(z)]^2\}^{-1/2}. \quad (1.23)$$

From (1.21) we have

$$\xi'(z) = A(2Az + vb^2)^{-1/2}. \quad (1.24)$$

Substituting (1.24) into (1.23), and going over to the variable  $y = z/h$  where  $h$  is defined by expression (1.22), we obtain

$$v_c = A[(x_0^2 + 2bx_0)y + b^2 + (A/v)^2]^{-1/2}. \quad (1.25)$$

Following [4], we define the limiting crystallization rate  $v_c^*$  as the limit of the value of  $v_c$  as  $k, v \rightarrow \infty$ . From (1.25) we have

$$v_c^* = A/x_0 y^{1/2}.$$

Thus, the values of  $v_c$  and  $v_c^*$  decrease monotonically along the crystallization surface. At the point  $y = 1$

$$v_c^* = v_{c1}^* = A/x_0. \quad (1.26)$$

Taking account of (1.26), relationship (1.25) can be represented in the form

$$v_c^* = v v_{c1}^* \{[(1 + 2b/x_0)y + (b/x_0)^2]v^2 + v_{c1}^{*2}\}^{-1/2}, \quad (1.27)$$

consequently, the value of  $v_c \rightarrow 0$ , both as  $k \rightarrow 0$  ( $b \rightarrow \infty$ ) and as  $v \rightarrow 0$ . At the point  $y = 0$ , as  $k \rightarrow \infty$  ( $b \rightarrow 0$ ),  $v_c \rightarrow v$ . Expression (1.27) for  $y = 1$  and as  $k \rightarrow \infty$  ( $b \rightarrow 0$ ) coincides in form with the formula of [4].

We give the results of a calculation of the process of solidification of an aluminum ingot with a thickness of  $5 \cdot 10^{-3}$  m at  $\Delta T = 620^\circ\text{C}$ ;  $\Delta T_{\text{sup}} = 40^\circ\text{C}$ ;  $\lambda_m = 208$ ;  $\kappa^* = 100$ ;  $\rho_c = 2.53 \cdot 10^3$ ; and of a titanium ingot with a thickness of 0.2 m at  $\Delta T = 1620^\circ\text{C}$ ;  $\Delta T_{\text{sup}} = 300^\circ\text{C}$ ;  $\lambda_m = 23$ ;  $\kappa^* = 122.17$ ;  $\rho_c = 4.2 \cdot 10^3$  [ $\lambda$ , kcal/(m · h · °K);  $\rho$ , kg/m<sup>3</sup>;  $\kappa$ , kcal/kg]. Figure 2 gives dependences of the relative depth of the liquid crater on the Pe number [dashed lines relate to the aluminum ingot: curve 1)  $Bi = \infty$ , 2)  $Bi = 0.464$ ; solid lines relate to the titanium ingot: curve 1)  $Bi = \infty$ , 2)  $Bi = 4.2$ ]. Figure 3 illustrates the change in the relative crystallization rate along the front of the solidification of aluminum and titanium ingots for different values of the Bi number (the numerical values and the designations of the curves are the same as in Fig. 2).

We note that the analysis made of the solution found was limited to taking account only of the first term of the series; however, the formulas obtained reflect with qualitative correctness the basic laws of the solidification process described, which follows from a comparison between them and known solutions [1, 2, 4]. In a control example, taking account of the second term of the series gave a correction in the second figure after the comma. Under these circumstances, in view of the nonlinearity of differential equation (1.19), the solution cannot be represented in analytical form and it is necessary to bring in numerical methods.

A connection between the temperature and the quantity  $\Theta(x, z)$  is established by the relationship (1.12). To obtain an explicit dependence of the function  $T$  on the variables  $x$  and  $z$ , the concrete dependence of the

thermal conductivity on the temperature must be given. Within a rather broad interval of change in the temperature, this dependence can be approximated by a linear law:

$$\lambda(T) = \mu + \omega T. \quad (1.28)$$

Substituting (1.28) into (1.12) and carrying out appropriate transformations, we obtain

$$T(x, z) = -\mu/\omega + \sqrt{(1 + \mu/\omega)^2 + 2\Theta(x, z)/\omega}, \quad (1.29)$$

where  $\Theta(x, z)$  is determined by the relationships (1.18) and (1.19). With an accuracy up to the first term of the series in (1.18), (1.19) we have

$$\Theta(x, z) = -\lambda_m \Delta T \left( 1 - \frac{\lambda_m / \text{Bi} + 1 - x}{\sqrt{A_1 z + (\lambda_m / \text{Bi})^2}} \right), \quad A_1 = 2\lambda_m \Delta T / \kappa^* \text{Pe}. \quad (1.30)$$

Thus, the temperature distribution in the solid phase of the ingot in the region  $0 \leq z \leq h$  will be determined by the relationships (1.29) and (1.30) with a linear law of the change of the thermal conductivity as a function of the temperature (1.28). In the general case, for a given dependence of  $\lambda$  on  $T$ , close to the real dependence, a polynomial of higher degree can be used or a piecewise-linear approximation can be adopted.

An important characteristic of the process of the crystallization of a continuous ingot is the rate of cooling at the crystallization front  $v_H = |\partial T / \partial t|_{x=\xi}$ , where the time  $t$  is defined in terms of the instantaneous coordinate  $z$  and the pulling rate by the relationship

$$t = z/v. \quad (1.31)$$

Bearing in mind that  $\partial \Theta / \partial z|_{x=\xi} = \partial T / \partial z|_{x=\xi}$ , from (1.18) we find

$$|\partial T / \partial z|_{x=\xi} = \kappa^* \text{Pe} [\xi'(z)]^2. \quad (1.32)$$

Or, returning to dimensional quantities in (1.32), and taking (1.24) and (1.31) into consideration, we obtain

$$v_T = \frac{\lambda_m^2 (T_c - T_{\text{li}})^2}{\kappa^* \rho_c \lambda_c (x_0 - \xi - \lambda_m/k)^2}, \quad (1.33)$$

where  $\xi(z)$  is defined by expression (1.20).

It follows from (1.33) that the rate of cooling at the crystallization front, with the above heat-transfer law, does not depend on the rate of pulling and is determined by three factors: the thickness of the ingot, the physical properties of the metal, and the cooling rate. The greatest value of the quantity under consideration,  $v_H$ , occurs at the point  $z=0$ , decreasing with the square of the thickness of the scale forming on the ingot:

$$\xi_{\text{eff}}(z) = x_0 - \xi(z) + \lambda_m/k.$$

From Eq. (1.32), taking account of (1.23) and (1.33), we obtain a relationship connecting the cooling rate at the crystallization front with the crystallization rate. In dimensional quantities we have

$$v_T = \frac{\kappa^* \rho_c}{\lambda_c} \frac{v^2 v_c^2}{v^2 - v_c^2}.$$

It is obvious that, for constancy of the value of  $v_H$  over the whole crystallization front, the crystallization rate must also be a constant quantity.

Below we obtain the cooling conditions for which the above quantities are constants. To this end, we take

$$\xi = 1 - y, \quad y = z/h, \quad (1.34)$$

assuming that the temperature of the cooling medium [or the  $\text{Bi}(y)$  number] is an unknown function of  $y$ . Substituting (1.34) into (1.18) and carrying out appropriate transformations, we obtain

$$\Theta(x, y) = \kappa^* a_m \sum_{i=1}^{\infty} \frac{1}{i!} [\beta(x-1+y)]^i, \quad \beta = \text{Pe}/h a_m.$$

The latter can be represented in the form

$$\Theta(x, y) = \kappa^* a_m \{1 - \exp[\beta(x-1+y)]\}. \quad (1.35)$$

Expression (1.29), taking account of (1.35), determines the temperature distribution in the scale forming on the ingot. Differentiating the function  $\Theta(x, y)$  from (1.35) with respect to  $x$ , we find the heat flux  $q(x, y)$  along the  $X$  axis at an arbitrary point  $(x, y)$  of the solid phase:

$$q(x, y) = \frac{\kappa^* \text{Pe}}{h} \exp[\beta(x-1+y)].$$

From this it follows that at the crystallization surface  $x=1-y$  the value of the heat flux  $q$  remains constant, which is in agreement with condition (1.15). From relationships (1.14)–(1.17), taking account of (1.34), we find an expression for the depth of the liquid crater:

$$h = \kappa^* Pe / q_0, \quad (1.36)$$

where  $q_0 = Bi(0)[1 - T_M(0)]$  is the value of the heat flux at the point of the start of crystallization ( $x=1, y=0$ ).

The distribution of the heat flux over the surface of the ingot ( $x=1$ ) has the form

$$q_1 = (\kappa^* Pe / h) e^{\beta y}.$$

From this it is apparent that the value of  $q_1(y)$  for the form of the liquid crater (1.34) is a monotonically rising function of the coordinate  $y$ . Consequently, the rate of external heat removal must rise along the direction of the motion of the ingot. Such cooling conditions can be assured by an increase in the heat-transfer coefficient or a lowering of the temperature of the cooling medium along the coordinate  $y$ . We note that in the case  $T_M = \text{const}$ ,  $Bi = \text{const}$ , the heat flux  $q_1(y)$  is a monotonically decreasing function of  $y$  [see (1.30)].

From condition (1.14), we determine the law of change in  $Bi(y)$  [or  $T_M(y)$ ] required for the removal of the heat flux  $q_1(y)$ . The expression for  $Bi(y)$  for a given value of the function  $T_M(y)$  has the form

$$Bi(y) = q_1 [1 + \alpha(1 - e^{\beta y}) - T_M(y)]^{-1}, \quad \alpha = \frac{\kappa^* a_m}{\lambda_m},$$

where  $h$  is determined by the relationship (1.36).

For a positive value of  $Bi(y)$  we must impose the condition

$$1 + \alpha(1 - e^{\beta y}) \geq T_M(y), \quad (1.37)$$

meaning that the temperature of the surface of the ingot cannot be lower than the temperature of the cooling medium. Substituting into (1.37) the expression for  $h$  and carrying out appropriate transformations, we obtain upper limits on the value of  $q_0$ , for which the solution under consideration exists:

$$q_0 \leq \kappa^* a_m \ln\{1 + \alpha^{-1}[1 - T_M(h)]\}.$$

The rate of crystallization  $v_c$  is determined from (1.23), taking account of (1.34) and (1.36):

$$v_c = v [1 + (\kappa^* Pe / q_0)^2]^{-1}.$$

For the dimensionless cooling rate at the crystallization front  $v_H$ , from (1.32), (1.34), and (1.36) we have

$$v_H = q_0^2 / \kappa^* Pe.$$

§2. The known solutions of the Stefan problem [2, 3, 5], as a rule, contain the assumption of the equality of the temperature of the melt over the whole volume of the liquid phase and the crystallization temperature. However, under real conditions, the temperature of the liquid metal exceeds the crystallization temperature by the amount of the superheat of the melt, which has an effect on the rates of solidification and the temperature distributions in the solid phase.

A rigorous solution of the problem of the solidification of an ingot in the presence of superheating of the melt is connected with the necessity of integrating the equation of thermal conductivity for the liquid and solid phases with the Stefan condition at the interface between them and it is necessary to bring in numerical methods of solution.

We shall give the derivation of the Stefan condition, approximately taking account of the heat flux due to superheating of the melt, for the case of the solidification of continuous ingots and ingots of limited volume.

In the case of a continuous ingot, we shall start from the same assumptions as above. Then the Stefan condition at the phase interface with superheating of the melt has the form

$$\left[ \lambda_2 \frac{\partial T_2}{\partial x} - \lambda_1 \frac{\partial T_1}{\partial x} \right]_{x=\xi(z)} = \kappa \rho_c v \xi'(z). \quad (2.1)$$

Here the subscripts 1 and 2 denote quantities relating to the liquid and solid phases, respectively. The remaining notation is the same as above.

The equation of the thermal conductivity for the liquid phase

$$\rho_1 C_1 v \frac{\partial T_1}{\partial z} = \frac{1}{x^v} \frac{\partial}{\partial x} \left( x^v \lambda_1 \frac{\partial T_1}{\partial x} \right)$$

is integrated with respect to  $x$  from 0 to  $\xi(z)$ , previously multiplying its left- and right-hand sides by  $x^\nu$ , where  $\nu = 0$  in the case of a flat ingot and  $\nu = 1$  in the case of a cylindrical ingot. Taking account of the condition of symmetry  $\partial T_1 / \partial x|_{x=0}$ , we obtain

$$\left[ \lambda_1 \frac{\partial T_1}{\partial x} \right]_{x=\xi(z)} = \frac{(C_1 \rho_1)_m \nu}{\xi^\nu} \int_0^\xi \frac{\partial T_1}{\partial z} x^\nu dx. \quad (2.2)$$

Here we take the value of  $C_1 \rho_1$  equal to its mean value in the interval of temperature change under discussion.

We represent the expression

$$\int_0^\xi \frac{\partial T_1}{\partial z} x^\nu dx$$

in the form

$$\int_0^\xi \frac{\partial T_1}{\partial z} x^\nu dx = \frac{\partial}{\partial z} \int_0^\xi T_1 x^\nu dx - T_C \xi^\nu \xi'. \quad (2.3)$$

Replacing the quantity  $T_1$ , standing under the integral sign in the right-hand side of expression (2.3), by its mean value in the volume of the liquid phase, we find

$$\frac{\partial}{\partial z} \int_0^\xi T_1 x^\nu dx = T_{1m} \xi^\nu \xi'. \quad (2.4)$$

Assuming further that  $T_{1m} = (T_0 + T_C)/2$ , where  $T_0$  is the temperature of the melt at the point  $z=0$ ,  $x=0$ , and substituting (2.4) into (2.3), we obtain

$$\int_0^\xi \frac{\partial T_1}{\partial z} x^\nu dx = (T_m - T_C) \xi^\nu \xi'. \quad (2.5)$$

Taking account of expressions (2.2) and (2.5), condition (2.1) assumes the form

$$\left[ \lambda_2 \frac{\partial T_2}{\partial x} \right]_{x=\xi(z)} = \rho_c v \left[ \kappa + \frac{(C_1 \rho_1)_m}{2\rho_c} (T_0 - T_C) \right] \xi'(z). \quad (2.6)$$

Introducing the effective value of the latent heat of fusion

$$\kappa^* = \kappa + \frac{(C_1 \rho_1)_m}{2\rho_c} (T_0 - T_C),$$

we represent relationship (2.6) in the form

$$[\lambda_2 \partial T_2 / \partial x]_{x=\xi(z)} = \rho_c v \kappa^* \xi'(z).$$

In the case of the solidification of a limited volume, the derivation will be analogous to the preceding, if, instead of the coordinate  $z$ , we introduce the time  $t = z/v$ .

Thus, taking account of the superheating of the melt is equivalent to an increase of the latent heat of fusion by an amount proportional to the superheating of the liquid phase with a coefficient of proportionality equal to  $(C_1 \rho_1)_m / 2\rho_c$ .

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